Int J Thermophys (2009) 30:1724–1732
DOI 10.1007/s10765-009-0621-3

Optimization of Irreversible Cogeneration Systems under Alternative Performance Criteria

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Received: 9 September 2008 / Accepted: 5 July 2009 / Published online: 22 July 2009

Abstract In this study, an exergy optimization has been performed for a cogeneration plant consisting of an irreversible Carnot heat engine. In the analysis, different objective functions have been defined based on alternative performance criteria and the optimum values of the design parameters of a cogeneration cycle were determined for different criteria. In this context, the effects of irreversibilities on the exergetic performance are investigated, and the results are discussed.

Keywords Cogeneration cycle · Irreversible Carnot · Performance criteria · Performance optimization

List of Symbols

$\dot{E}$ Exergy rate
$\dot{Q}$ Rate of heat transfer
I Irreversibility parameter
R Power to process heat ratio
S Entropy
T Temperature
$\dot{W}$ Power generated from the cogeneration system

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α, β, γ	Thermal conductances in the heat source, heat sink and heat consumer sides, respectively
η	Efficiency
ϕ	Ratio of heat-source to heat-sink temperature
Ψ	Ratio of heat-consumer to heat-sink temperature

Subscripts

H	Heat source
L	Heat sink
K	Heat consumer
max	Maximum
X	Warm working fluid
Y	Cold working fluid
Z	Process working fluid
T	Total

Superscripts

–	Dimensionless
*	Optimum

1 Introduction

Cogeneration plants, in which heat and power are produced together, are widely used as they are more advantageous in terms of energy and exergy efficiencies than plants which produce heat and power separately [1]. Recently, numerous studies have been carried out on performance analysis of cogeneration systems [2–7]. Usually, in these studies, different performance criteria, such as exergy efficiency and artificial thermal efficiency, are investigated.

Yilmaz [4] examined performance analysis based on compared performance criteria about the irreversible Carnot cycle modified for a cogeneration system. Performance analysis based on compared performance criteria about cogeneration plants including irreversibilities was not found in the literature.

In this study, an irreversible Carnot cycle is modified for cogeneration with external and internal irreversibilities. The effects of the ratio of power to heat demanded by the heat consumer process, extreme temperatures, heat consumer temperature, and irreversibilities on the global and optimal performances have been determined and discussed.

2 Theoretical Model

The irreversible Carnot cycle model for a cogeneration system and its T–S diagram are given in Fig. 1 [8]. In the model, internal and external irreversibilities are consid-

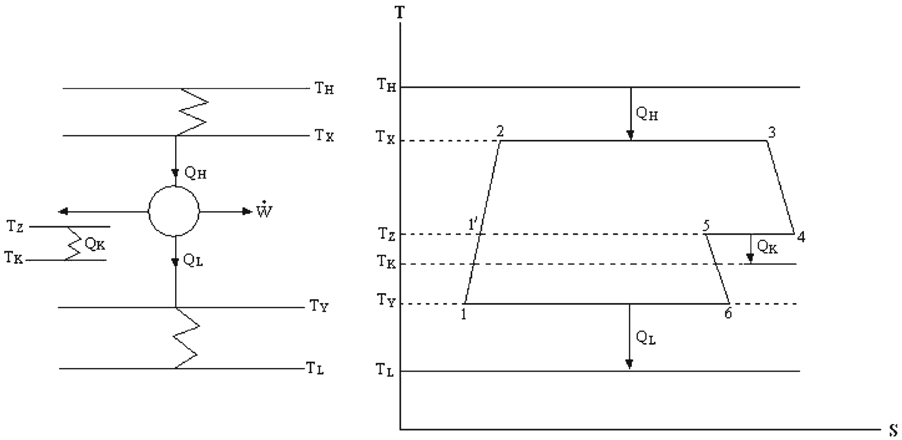

Fig. 1 Irreversible cogeneration cycle and its *T–S* diagram

ered. The given cogeneration cycle model operates between three heat reservoirs of temperatures T_H, T_L, and T_K. The temperatures of the working fluids exchanging heat with the reservoirs at T_H, T_L, and T_K are T_X, T_Y, and T_Z, respectively. In the cycle, $\dot{Q}_H$ is the rate of heat transfer from the heat source at temperature T_H to the warm working fluid at constant temperature T_X in process 2–3, $\dot{Q}_L$ is the rate of heat transfer from the cold working fluid at constant temperature T_Y to the heat sink at temperature T_L in process 6–1, and $\dot{Q}_K$ is the rate of heat transfer from the working fluid at constant temperature T_Z to the heat consuming device at temperature T_K in process 4–5.

The model given in Fig. 1 is an appropriate model for cogeneration plants in practice which have pass-out condensing steam turbines. In these plants, some steam is extracted from turbine at an intermediate pressure and transferred to a heat consumer for the heating process. Then, the condensate is returned to the plant. This operation is equivalent to process 4–5 in the given model for a closed system with the only exception that the heat transfer takes place without mass transfer.

When the heat transfer obeys Newton's law, we can write [9–11]

$$\dot{Q}_H = \alpha(T_H - T_X), \tag{1}$$
$$\dot{Q}_L = \beta(T_Y - T_L), \tag{2}$$
$$\dot{Q}_K = \gamma(T_Z - T_K), \tag{3}$$

where α, β, and γ are the thermal conductances of the heat source, heat sink, and heat consumer sides, respectively. The power output of the cogeneration cycle according to the first law is [12]

$$\dot{W} = \dot{Q}_H - \dot{Q}_L - \dot{Q}_K. \tag{4}$$

Substituting Eqs. 1, 2, and 3 into Eq. 4, the power output becomes

$$\dot{W} = \alpha(T_H - T_X) - \beta(T_Y - T_L) - \gamma(T_Z - T_K). \tag{5}$$

The exergy rate of power output is

$$\dot{E}_W = \dot{W}, \tag{6}$$

and the exergy rate of process heat is

$$\dot{E}_Q = \dot{Q}_{\mathrm{K}} \left(1 - T_{\mathrm{L}}/T_{\mathrm{Z}}\right), \tag{7}$$

where T_{L} in this equation is thought to be the environment temperature. Then, the total exergy rate delivered by the cogeneration system is

$$\dot{E}_{\mathrm{T}} = \dot{E}_W + \dot{E}_Q. \tag{8}$$

By using Eqs. 3, 5, 6, and 7 in Eq. 8, it becomes

$$\dot{E}_{\mathrm{T}} = \alpha(T_{\mathrm{H}} - T_{\mathrm{X}}) - \beta(T_{\mathrm{Y}} - T_{\mathrm{L}}) - \gamma(T_{\mathrm{Z}} - T_{\mathrm{K}})T_{\mathrm{L}}/T_{\mathrm{Z}}. \tag{9}$$

Due to internal dissipations of the working fluid, all branches of the cycle are irreversible. For an irreversible system, the second law of thermodynamics requires that

$$\oint \frac{\delta Q}{T} = \frac{Q_{\mathrm{H}}}{T_{\mathrm{X}}} - \frac{Q_{\mathrm{L}}}{T_{\mathrm{Y}}} - \frac{Q_{\mathrm{K}}}{T_{\mathrm{Z}}} < 0 \tag{10}$$

If irreversible parameters are adopted as I_1 and I_2, we obtain

$$I \frac{Q_{\mathrm{H}}}{T_{\mathrm{X}}} - \left(\frac{Q_{\mathrm{L}}}{T_{\mathrm{Y}}} + \frac{Q_{\mathrm{K}}}{T_{\mathrm{Z}}}\right) = 0 \tag{11}$$

where irreversibility parameters I_1 and I_2 are defined as

$$I = \frac{(S_6 - S_1) + (S_4 - S_5)}{S_3 - S_2} = \frac{Q_{\mathrm{L}} T_{\mathrm{X}}}{Q_{\mathrm{H}} T_{\mathrm{Y}}} + \frac{Q_{\mathrm{K}} T_{\mathrm{X}}}{Q_{\mathrm{H}} T_{\mathrm{Z}}} > 1 \tag{12}$$

which characterize the degree of irreversibilities. Substituting Eqs. 1, 2, and 3 into Eq. 11, we obtain the following equation:

$$I\alpha(T_{\mathrm{H}} - T_{\mathrm{X}})/T_{\mathrm{X}} - \beta(T_{\mathrm{Y}} - T_{\mathrm{L}})/T_{\mathrm{Y}} - \gamma(T_{\mathrm{Z}} - T_{\mathrm{K}})/T_{\mathrm{Z}} = 0. \tag{13}$$

Equation 13 shows clearly that when $I = 1$, the system is reversible; when $I > 1$, the system is internally irreversible.

We also require that the ratio of power to the heat demanded by the heat consumer is known and constant, i.e.,

$$\frac{\dot{W}}{Q_{\mathrm{K}}}\left[\alpha(T_{\mathrm{H}} - T_{\mathrm{X}}) - \beta(T_{\mathrm{Y}} - T_{\mathrm{L}}) - \gamma(T_{\mathrm{Z}} - T_{\mathrm{K}})\right]/\gamma\,(T_{\mathrm{Z}} - T_{\mathrm{K}}) = R. \tag{14}$$

Using the above definitions alternative performance criteria can be defined as follows:

2.1 Energy Utilization Factor [3, 4]

The energy utilization factor (*EUF*) can be defined by using Eqs. 1, 3, and 4 as

$$EUF = \frac{\dot{W} + \dot{Q}_{\mathrm{K}}}{\dot{Q}_{\mathrm{H}}} = 1 - \frac{\beta(T_{\mathrm{Y}} - T_{\mathrm{L}})}{\alpha(T_{\mathrm{H}} - T_{\mathrm{X}})} \tag{15}$$

The *EUF* has the advantage of simplicity. However, the *EUF* cannot discriminate the quality difference between work and heat.

2.2 Artificial Thermal Efficiency [3, 4]

An alternative performance criterion is the artificial thermal efficiency (η_{A}). The given energy to the cogeneration plant is assumed to be reduced by the consuming heat rate ($\dot{Q}_{\mathrm{K}}$). The thermal efficiency is then given by

$$\eta_{\mathrm{A}} = \frac{\dot{W}}{\dot{Q}_{\mathrm{H}} - \dot{Q}_{\mathrm{K}}} = 1 - \frac{\beta(T_{\mathrm{Y}} - T_{\mathrm{L}})}{\alpha(T_{\mathrm{H}} - T_{\mathrm{X}}) - \gamma(T_{\mathrm{Z}} - T_{\mathrm{K}})} \tag{16}$$

2.3 Exergy Efficiency [2–4]

The exergy efficiency describes the quality difference of energy with the parameter exergy as this criterion is more reasonable. The exergy efficiency (η_{E}) can be defined as

$$\eta_{\mathrm{E}} = \frac{\dot{E}_{\mathrm{T}}}{\dot{Q}_{\mathrm{H}}(1 - T_{\mathrm{L}}/T_{\mathrm{H}})} = \frac{\alpha(T_{\mathrm{H}} - T_{\mathrm{X}}) - \beta(T_{\mathrm{Y}} - T_{\mathrm{L}}) - \gamma(T_{\mathrm{Z}} - T_{\mathrm{K}})T_{\mathrm{L}}/T_{\mathrm{Z}}}{\alpha(T_{\mathrm{H}} - T_{\mathrm{X}})(1 - T_{\mathrm{L}}/T_{\mathrm{H}})} \tag{17}$$

It is possible to optimize the total exergy output given in Eq. 9, the energy utilization factor given in Eq. 15, the artificial thermal efficiency given in Eq. 16, and the exergy efficiency given in Eq. 17 with respect to T_{X}, T_{Y}, and T_{Z} using constraints given in Eqs. 13 and 14. This optimization has been carried out numerically, and the results are discussed in the next section.

3 Performance Analysis

The results for a reversible case ($I = 1$) presented in this article replicate the results given by Yilmaz [4] in which a reversible Carnot cycle modified for a cogeneration system with external irreversibilities was examined. The variations of the normalized total exergy rate ($\overline{\dot{E}} = \dot{E}_{\mathrm{T}}/\alpha T_{\mathrm{L}}$) versus EUF, η_{A}, and η_{E} for $I = 1.1$ and various R (0.1, 0.5, 1, 2, 4, 20) are presented in Fig. 2, and it can be seen that the energy utilization factor at maximum exergy (*EUF**) and exergy efficiency at maximum exergy

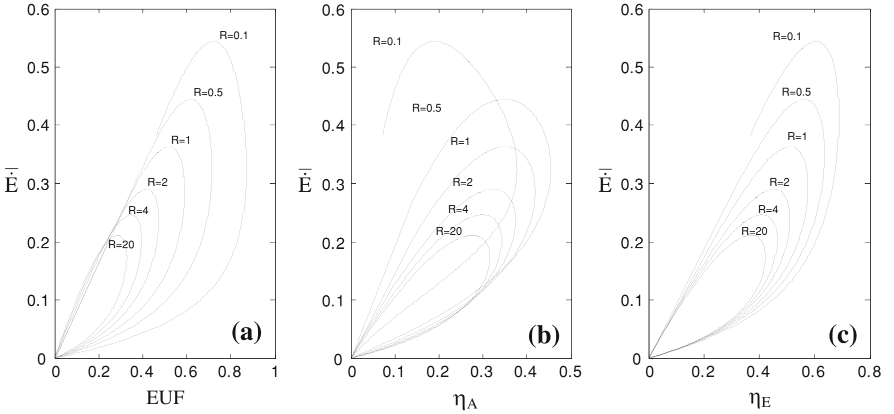

Fig. 2 Variations of normalized total exergy with respect to (**a**) *EUF*, (**b**) η_A, and (**c**) η_E for different R values ($\psi = 1.67, \phi = 4, I = 1.1, \alpha = \beta = \gamma$)

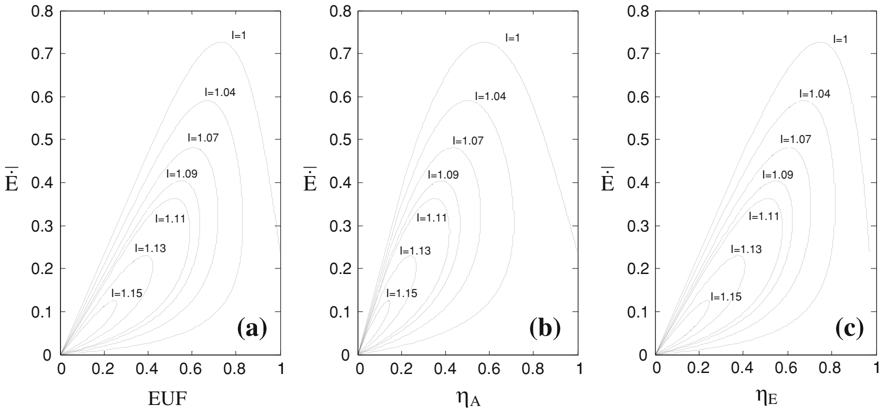

Fig. 3 Variations of normalized total exergy with respect to (**a**) *EUF*, (**b**) η_A, and (**c**) η_E for different I values ($\psi = 1.67, \phi = 4, R = 1, \alpha = \beta = \gamma$)

(η_E^*) increase with decreasing R; however, the artificial thermal efficiency at maximum exergy (η_A^*) increases until $R = 1$, which then gradually decreases after $R = 1$.

The variations of the normalized total exergy rate with respect to *EUF*, η_A, and η_E for $R = 1$ and various irreversibility values ($I = 1$, 1.04, 1.07, 1.09, 1.11, 1.13, 1.15) are presented in Fig. 3. From these plots, it can be seen that the global performances and the maximum values of the total exergy rate ($\overline{\dot{E}}_{max}$) decrease with increasing I value which corresponds to increasing irreversibility.

Variations of the optimal performance criteria (EUF^*, η_A^*, η_E^*) with respect to the extreme temperature ratio ($\phi = T_H/T_L$) for different irreversibility values ($I = 1$, 1.05, 1.11, and 1.15) are shown in Fig. 4. As can be seen from these plots, the optimal performance criteria increase with an increase in the extreme temperature ratio. It is also possible to say, that for small ϕ values, the irreversibilities have more influence

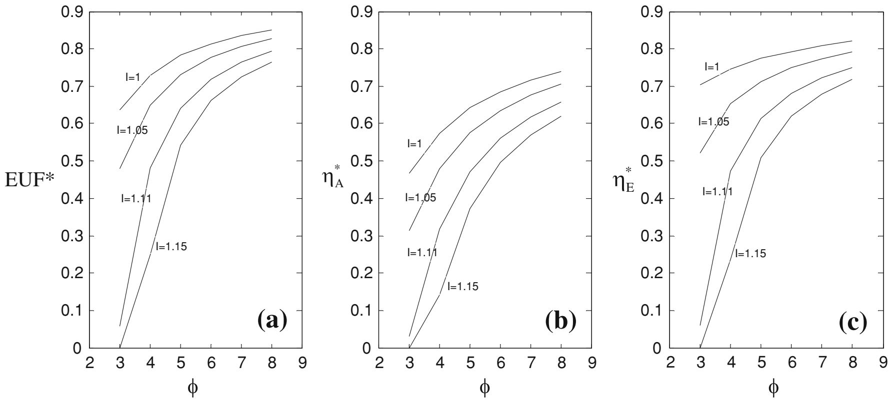

Fig. 4 Variations of the optimal performance criteria: (**a**) *EUF*, (**b**) η_A, and (**c**) η_E with respect to ϕ for different I values ($\psi = 1.67$, $R = 1, \alpha = \beta = \gamma$)

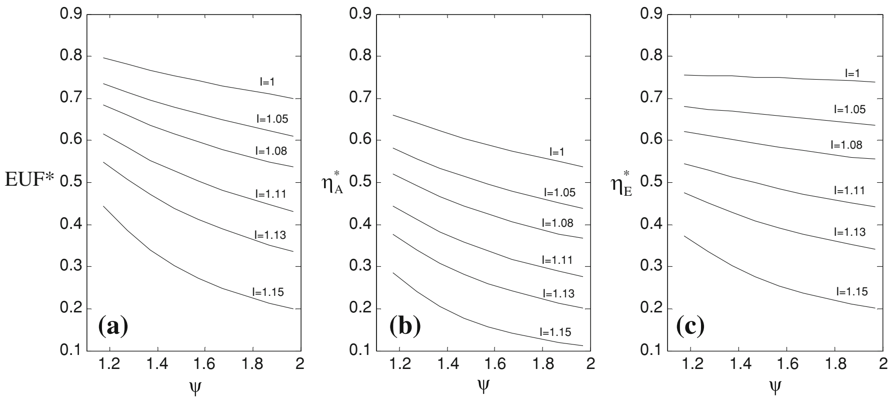

Fig. 5 Variations of the optimal performance criteria: (**a**) *EUF*, (**b**) η_A, and (**c**) η_E with respect to ψ for different I values ($\phi = 4$, $R = 1, \alpha = \beta = \gamma$)

on the performance, and so it is more important to take precautions to irreversibilities for small extreme temperature ratios.

In Fig. 5, the variations of optimal performance criteria are plotted with respect to the process heat temperature ratio ($\psi = T_K/T_L$) for different I values. It can be observed from these plots that as the process heat temperature ratio increases, EUF^*, η_A^*, and η_E^* gradually decrease. It is also possible to conclude from these plots that if the consuming temperature increases, the irreversibilities have more influence on the performance.

Figure 6 shows variations of EUF^*, η_A^*, and η_E^* versus R for different irreversibility values. According to the plots, the performance criteria investigation can be divided into two different regions $0 < R < 1$ and $1 < R < \infty$. For the first region ($0 < R < 1$), the evolution of the artificial thermal efficiency at the maximum exergy rate is more critical than the other criteria. The maximum value of the optimal artificial thermal efficiency will be carried out for $R = 1$. As R increases, the values of

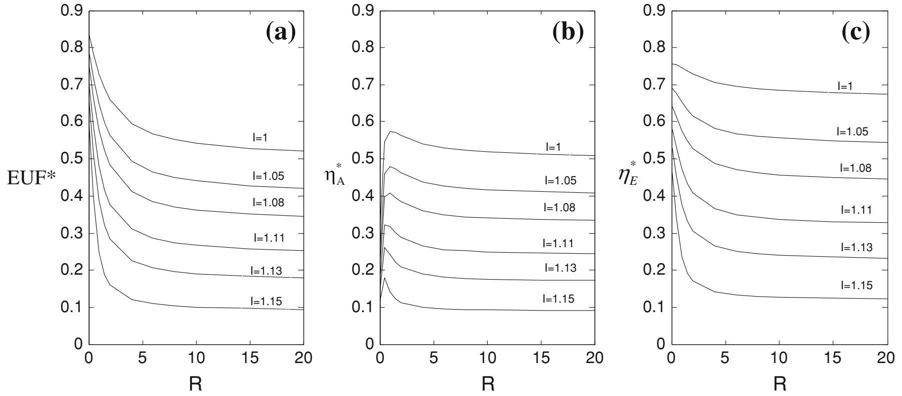

Fig. 6 Variations of the optimal performance criteria: (**a**) *EUF*, (**b**) η_{A}, and (**c**) η_{E} with respect to R for different I values ($\phi = 4$, $\psi = 1.67$, $\alpha = \beta = \gamma$)

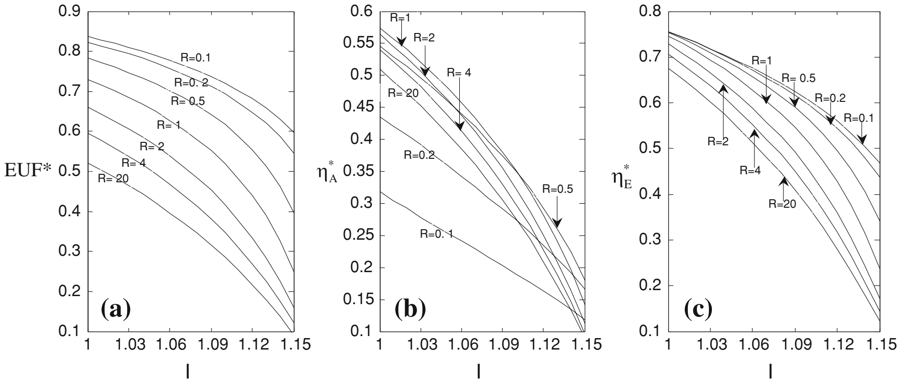

Fig. 7 Variations of the optimal performance criteria: (**a**) *EUF*, (**b**) η_{A}, and (**c**) η_{E} with respect to I for different R values ($\phi = 4$, $\psi = 1.67$, $\alpha = \beta = \gamma$)

the performance criteria get closer together, and for the $R \to \infty$ limiting case, they reach the same value. This value is the thermal efficiency at maximum power output for an irreversible heat engine [13,14], ($\eta = 1 - \sqrt{I \frac{T_{\mathrm{L}}}{T_{\mathrm{H}}}}$). It can also be observed from Fig. 6 that the EUF at maximum exergy rate is always greater than the artificial thermal efficiency and the exergy efficiency at the maximum exergy rate for the same irreversibility values.

4 Conclusion

A performance analysis has been carried out for an irreversible Carnot cycle modified for a cogeneration system with external and internal irreversibilities. The results for the reversible case of this study ($I = 1$) are the same as the results given by Yilmaz [4]. The effects of irreversibility have been examined for different operating conditions. Performance criteria have been evaluated, and the results discussed. As the irreversibilities increase, that is, as I increases, the general and optimal performances gradually decrease. It is also concluded that for small ϕ values and the large ψ values the irreversibilities have more effectual on the performance, so it is more important to take precautions for reducing irreversibilities. Finally, it is seen that $R = 1$ is a critical value for the optimal artificial thermal efficiency, and it should be evaluated separately for $R < 1$ and $R > 1$.

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